A KALDORIAN MACROECONOMIC INDEX OF ECONOMIC WELFARE[1]

René A. Medrano-B[2]
Joanílio Rodolpho Teixeira[3]

ABSTRACT
The international crisis, started in 2008, generated some skepticism on the world economists since it was not anticipated the magnitude and impact of the disturbance. This work deals with a tool that helps to measure, numerically, the performance of an economy as well as to compare the impact of a crisis among nations and/or regions. We start from the Kaldor's (1971) view which led to the construction of a diagram known as the "Magic Square" – MS by taking into account sensitive parameters of a country's economy. The original idea of the MS offered only a visual perspective. Here we extend the idea through a mathematical analysis that quantifies the geometry of the Square. As a result we end up with a formal indicator, here called Index of Economic Welfare, that intends to capture the economic welfare of a country. As an application of the index we use recent economic data of Brazil and Chile. The results show the impact of the crisis in both nations.

Key words
Kaldor, Magical Square, Crisis, Brazil, Chile.

JEL Classification
F34, G01, G18, B, E12

[1] We thank Cristiane Soares and Danielle Sandi for the collaboration. This research is supported by the National Council of Science and Technology (CNPq/Brazil). E-mails: renemedrano@hotmail.com and joanilioteixeira@hotmail.com.
[2] Instituto Nacional de Pesquisas Espaciais (INPE), S.J. dos Campos, SP.
[3] Universidade de Brasília-UnB, Brasília, DF.

1. INTRODUCTION

The analysis of how economic crisis influences a country's economic performance has received renewed interest nowadays, due to the 2008 financial crash which started in the USA and spread worldwide. Such disturbance is still hitting many nations abroad. It also led to some skepticism on the advice of professional economists and well known research centers. They did not anticipate the magnitude of the crisis and its real impact, which turned out to be more than just financial. Furthermore, the orthodox macroeconomic theory and applications was unable to suggest proper policies to by-pass a number of real troubles still hitting many regions.

The question is how to evaluate the economic performance of a nation along a crisis, or to compare the impact of the crisis among countries. One way is to take a view into the seminal article of Kaldor (1971) where the author assessed the macroeconomic performance of Britain. He introduced an analytical instrument to deal with conflicts in national policy objectives. His approach is based upon four economic criteria expressed in quantitative terms, as targets: full employment, balance of payments, rate of growth and wage-increase or income policy.

However, Kaldor's paper does not include neither a mathematical analysis nor a single diagram. Latter, Karl Schiller (a German scientist and politician leader of the Social Democratic Party from 1966 to 1972) introduced a graphical representation of Kaldor's idea. The resulting intuitive diagram allowed a visual diagnosis of the macroeconomic evolution of a country. Such an instrument was called "Magic Square" – MS. Soon after, economists at the OCDE began using this tool with a minor modification (instead of Policy Objectives, Inflation was used) but otherwise preserving the fundamental Kaldor's idea. Bernard et. Alli (1988) indicated a basic illustration of the MS.

In this paper we extend the geometric view of the MS to a numerical evaluation of a modified Magic Square area, which could not be calculated otherwise. As a consequence of the analysis an indicator of Economic Welfare is introduced. As an application we use economic data of Brazil and Chile. The results show the impact of the crisis in both countries.

2. "MAGIC SQUARE" REVISITED

The Magic Square has been conceived in such a way that its four directions (N, S, E and W) were aligned with Growth, Inflation, Trade and Unemployment indexes, respectively. All four directions are at different scales, even though expressed in percentages. In this conception, economists were able to take a quick look to the evolution of the economic situation of a country by joining adjacent indexes with straight lines. The state of the economy would then be related to the size "area" of the resulting quadrangle. Of course, the area of the quadrangle cannot be calculated, because of the non-uniform scales of the axes. Nonetheless, full acceptance had the concept of the ideal, wonderland, economy represented by the larger area of the quadrangle, which in fact was drawn as a lozenge with four right angles, therefore the name of Magic Square. In this section we re-define de axes of the MS as to make them uniform and make possible the computation of the quadrangle area.

In order to accomplish this task we define the domains of the original economic indexes. These bounding conditions are given by historical circumstances representing a kind of "wonderland macroeconomic configuration". Let:

$$0 \leq \gamma \leq 10 \qquad -2 \leq \tau \leq 4 \qquad 10 \geq \varphi \geq 0 \qquad 12 \geq \zeta \geq 0, \tag{1}$$

where, γ stands for Growth, τ for Trade, φ for Inflation and ζ for Unemployment. In the new MS, all four scales we arbitrary defined to be uniform from 0 to a, where a is a numerical constant to be evaluated by normalizing to a **unit area** the modified MS. So, the new corresponding indexes are:

$$0 \leq \gamma' \leq a \qquad 0 \leq \tau' \leq a \qquad 0 \leq \varphi' \leq a \qquad 0 \leq \zeta' \leq a. \tag{2}$$

Notice that, by this definition, we conceive a perfect square with uniform axes. The next task is to find the transformation relations from the un-primed to the primed variables.

Since all original variables - economic indexes - have linear scales, the new ones should also be linear. As an example, let us take the transformation of τ and then of φ, which may seem to be harder to visualize.

Trade τ:

τ |----------|
-2 4

τ' |----------|
0 a

FIGURE 1

Of course, to any point in the τ axis will correspond, linearly, a τ' on the other. This is just what is found in a straight-line representation, in an orthogonal coordinate system. So, taking τ in the x-axis and τ' in the y-axis, the straight line is shown in Fig. 2. The problem reduces to the finding of the straight line equation that goes through the points $(-2, 0)$ and $(4, a)$. The result:

$$\tau' = \frac{a}{6}(\tau + 2) \qquad (3)$$

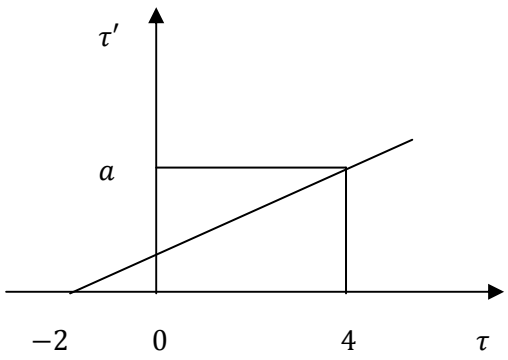

FIGURE 2

Inflation φ.

φ |----------|
10 0

φ' |----------|
0 a

FIGURE 3

Likewise, the straight line equation that goes through the points $(10, 0)$ and $(0, a)$ is:

$$\varphi' = \frac{a}{10}(10 - \varphi) \quad (4)$$

In the same fashion we can find the scale transformation of the other two variables:

$$\gamma' = \frac{a}{10}\gamma \quad (5)$$

$$\varsigma' = \frac{a}{12}(12 - \varsigma) \quad (6)$$

Now, the modified MS (we would like to say, the 'primed' MS) is shown in Fig. 4.

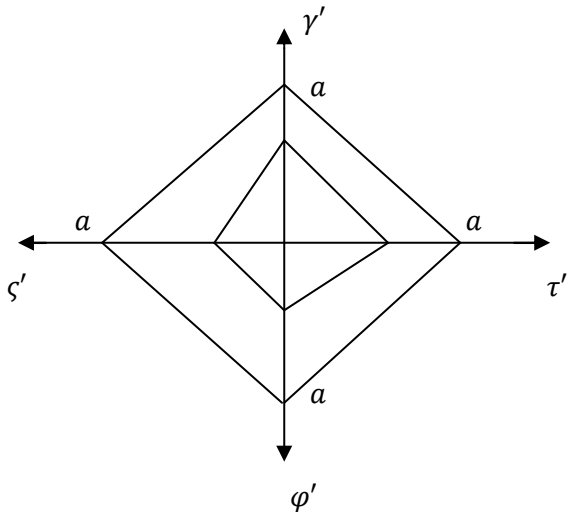

FIGURE 4

This figure shows the wonderland 'primed' MS, represented by the largest lozenge, which is a truly square figure rotated in an angle of 45°. Now we can find the value of a by defining the area A'_W, corresponding to the wonderland economy, to be 1. Therefore,

$$A'_W = 4 \times \frac{1}{2}a^2 = 1$$

So,

$$a^2 = \frac{1}{2} \quad (7)$$

For arbitrary values of γ, τ, φ and ζ, the real world economy is represented by the innermost quadrangle in Fig. 4. The corresponding area A', of the quadrangle, is just the addition of all four rectangular triangles:

$$A' = \frac{1}{2}(\tau'\gamma' + \tau'\varphi' + \zeta'\varphi' + \zeta'\gamma') \quad (8)$$

At this point we remember that the original idea of the MS was to allow a quick look to the state of the economy situation of a country. In fact, by looking to the evolution of the size "area" of the quadrangle, one could know the evolution of the "health" of the economy of a country. Equation (8) is exactly the numerical value of the surface area we looked for.

Furthermore, substituting all values of the primed variables (γ', τ', φ' and ζ'), from Eqs. (3) through (6), into A' of Eq. (8), we have:

$$240A' = (\tau + 2)\gamma + (\tau + 2)(10 - \varphi) + \frac{1}{2}(12 - \zeta)(10 - \varphi) + \frac{1}{2}(12 - \zeta)\gamma \quad (9)$$

With these considerations, and based on Eq. (9), we introduce a new economy index:

$\boldsymbol{\epsilon}$ – **Index of Economic Welfare**, given by the equation:

$$240\boldsymbol{\epsilon} = (\tau + 2)\gamma + (\tau + 2)(10 - \varphi) + \frac{1}{2}(12 - \zeta)(10 - \varphi) + \frac{1}{2}(12 - \zeta)\gamma \quad (10)$$

It can be verified that the range values of $\boldsymbol{\epsilon}$ are given by:

$$0 \leq \epsilon \leq 1, \quad (11)$$

where 0 corresponds to a catastrophic economy of a country (Growth $\gamma = 0$, Trade $\tau = -2$, Inflation $\varphi = 10$ and Unemployment $\zeta = 12$) whereas 1 (or 100%) portraits the wonderland economy ($\gamma = 10, \tau = 4, \varphi = 0$ and $\zeta = 0$).

3. THE CASE OF BRAZIL AND CHILE

As an application we use available data of the current world crisis (triggered by the financial crush of 2008 in USA). Table 1 (Source: IBGE and IMF Annual Report, 2011) shows the values used in this work before the actual crisis, 2004-2007, and during the crisis, 2008-2011, of both countries.

TABLE 1

Economic Variables	BRAZIL		CHILE	
	2004-2007	2008-2011	2004-2007	2008-2011
γ	4,7	3,7	5,2	3,3
τ	1,2	-1,9	3,2	0,4
φ	5,3	5,6	3,0	3,8
ς	8,7	5,5	7,9	6,4

Substituting these values into Eq. (10) we end up with the following numerical values for index $\boldsymbol{\epsilon}$, for both countries in the same time intervals:

$\boldsymbol{\epsilon}$	**19.0%**	**11.3%**	**36.8%**	**20.6%**

These numerical results, of the now called index of economic welfare, reveal that Chile's economic indicator was better than Brazil's before and during the crisis. Nonetheless, when we compare the rate of change of the index $\frac{\Delta\epsilon}{\Delta t}$ (where $\Delta t = \frac{2008-2011}{2} - \frac{2007-2004}{2} =$ 4 years) in these two time intervals, we notice that the Brazilian economic welfare was less affected (-1.93%/4 years) than the Chilean's (-4.05%/4 years).

4. CONCLUDING REMARKS

The main purpose of this work was to make possible the numerical evaluation of the economy performance of a country, based on a new approach to the Magical Square, originally conceived by Karl Schiller - with latter contributions from the OCDE – otherwise preserving the conceptual framework derived from Kaldor (1971) seminal work. The result of the analysis led us to define a new index, $\boldsymbol{\epsilon}$, here called Index of Economic Welfare. It should be stressed, however, that the numerical value of $\boldsymbol{\epsilon}$ depends upon the bounding limits - within each domain - of the economic indexes, as it is clear in Equations (1) and (10). Of course, this is a limitation as for its use, and comparison, among other economies and times. This is to say, the catastrophic economy, defined by $\boldsymbol{\epsilon} = 0$, as well as the Wonderland economy ($\boldsymbol{\epsilon} = 1$) are valid only for bounding values chosen in this work. An approach to derive a more general expression for $\boldsymbol{\epsilon}$, such as to avoid limitations, is now in progress.

As an application, a formal approach to the economies of Brazil and Chile, primarily due to the availability of data, was performed. By this approach, it was

possible to establish that the current world crisis was hitting harder to Chile than to Brazil, during the period of data availability (2008-2011) in spite of the comparatively better economic health indexes of the former.

REFERENCES

Bernard, M. ; Drouet, M. ; Echaudemaison, C. & Pinet, N. (1988) Economie et Société Française. Editions Nathan, Paris, France.

Kaldor, N. (1971) Conflicts in National Economic Objectives. The Economic Journal. 81 (321), pp. 1-16.